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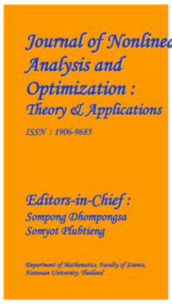


# VERTEX REGULAR AND IRREGULAR FUZZY SIGNED GRAPH

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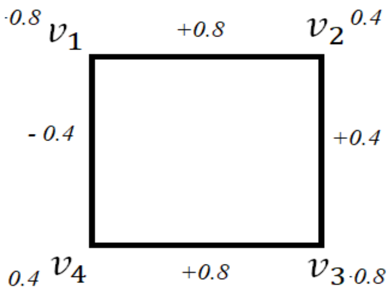
**ABSTRACT**
In particular, we present vertex regular and irregular fuzzy signed graphs and their detailed properties, including necessary and sufficient conditions for a vertex regularity in fuzzy signed graphs. In this paper, we explore the concept of vertex regular and irregular fuzzy graphs, extending fuzzy graph to fuzzy signed graphs, which are characterized by vertices and edges associated with a fuzzy membership function and sign values ( + ve and - ve).
**Keywords:** Fuzzy signed graph, Regular fuzzy signed graph, Totally vertex regular fuzzy signed graph, Irregular fuzzy signed graph.

## 1. VERTEX REGULAR FUZZY SIGNED GRAPH

If each vertex in a fuzzy signed graph Γ has the same degree, the graph is said to be regular.

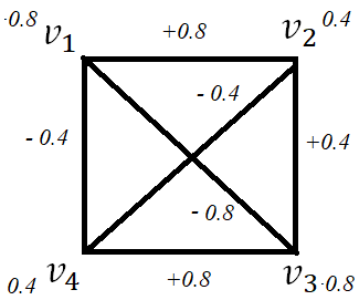
$$d(v_1) = 0.8 - 0.4 = 0.4$$
$$d(v_2) = 0.8 - 0.4 = 0.4$$
$$d(v_3) = 0.8 - 0.4 = 0.4$$
$$d(v_4) = 0.8 - 0.4 = 0.4$$




## 2. TOTALLY VERTEX REGULAR FUZZY SIGNED GRAPH

When every vertex in Γ(υ,δ,μ) has the same total degree, let's say k, then Γ is said to be either k fully regular FSG or totally vertex regular FSG of total degree k.

$$d(v_1) = 0.8 - 0.4 - 0.8 + 0.8 = 0.4$$
$$d(v_2) = 0.8 - 0.4 + 0.4 - 0.4 = 0.4$$
$$d(v_3) = 0.8 - 0.4 - 0.8 + 0.8 = 0.4$$
$$d(v_4) = 0.8 - 0.4 - 0.4 + 0.4 = 0.4$$



**Theorem:**

Let $\Gamma = (v, \delta, \mu)$ be a FSG and $\mu$ be a constant function then $\Gamma$ is regular if and only if $\Gamma$ if TRFSG.

**Proof:**

Let $\Gamma = (v, \delta, \mu)$ be a RFSG . Therefore $\deg(u) = k$ for all $u \in v$ where k is constant.
Now to show that $T\deg(u) = constant$ for all $u \in v$.
Now $t\deg u = \deg(u) + \mu(u)$
$= k + c$ since $\mu(u) = c$.
This is true for every $u \in v$.
Hence $\Gamma$ is TRFSG.

**Conversely,**

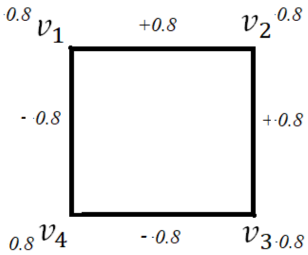
Let $\Gamma$ be a TRFSG then $t\deg(u)$ is constant and let it be $P$ for all $u \in v$.
We need to prove $\deg(u) = constant$ for every $u \in v$.
Now $t\deg(u) = \deg(u) + \mu(u)$
$\mathrm{P} = \deg(u) + C$
$\deg(u) = P - C \Longrightarrow a\ constant$ for every $u \in v$.
Therefore $\Gamma$ is Regular.

**Theorem:**

If $\Gamma = (v, \delta, \mu)$ be a RFSG and TRFSG then $\mu$ be a constant function.

**Proof:**

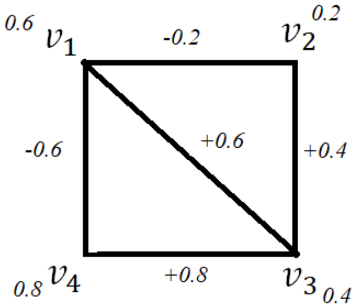
Since $\Gamma$ is a RFSG therefore $\deg(u) = P_1\ (say)$ for every $u \in v$.
Again $\Gamma$ is a TRFSG therefore $\mathrm{T}\deg(u) = P_2\ (say)$ for every $u \in v$.
Now $t\deg u = \deg(u) + \mu(u)$
$P_2 = P_1 + \mu(u)$
$P_2 - P_1 = \mu(u)$ for every $u \in v$.
$\mu(u)$ is constant for all $u \in v$.
Hence $\mu$ is constant function.
The converse part of the theorem is not necessary true . This is justified by example.




$\mu(u) = 0.8$
$\mu(v_1, v_2) = 0.8$ $\mu(v_2, v_3) = 0.8$
$\mu(v_3, v_4) = 0.8$ $\mu(v_4, v_1) = 0.8$
$deg(v_1) = 0.8 - 0.8 = 0$ $t\ deg(v_1) = 0.8$
$deg(v_2) = 0.8 + 0.8 = 1.6$ $t\ deg(v_2) = 1.6 + 0.8 = 2.4$
$deg(v_3) = 0.8 - 0.8 = 0$ $t\ deg(v_3) = 0.8$
$deg(v_4) = -0.8 - 0.8 = -1.6$ $t\ deg(v_4) = -1.6 + 0.8 = -0.8$
This shows that $\Gamma$ is neither RFSG and TRFSG.
Therefore $\mu$ is constant.

## 3. IRREGULAR FUZZY SIGNED GRAPH

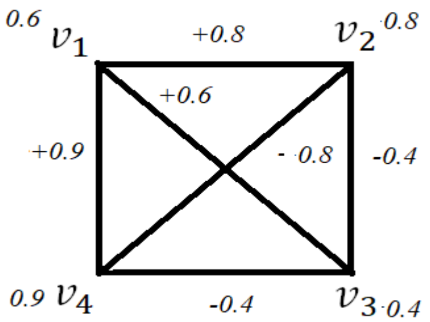
A FSG is said to vertex irregular if there is a vertex has distinct degree.




$deg(v_1) = 0.6 - (0.2 + 0.6) = 0.6 - 0.8 = -0.2$
$deg(v_2) = 0.4 - 0.2 = 0.2$
$deg(v_3) = 0.4 + 0.8 + 0.6 - 0 = 1.8$
$deg(v_4) = 0.8 - 0.6 = 0.2$

## 4. NEIGHBOURLY VERTEX IRREGULAR FUZZY SIGNED GRAPH

When every two adjacent vertices in a fuzzy signed graph Γ have different degrees, the graph is referred to as a neighborly vertex irregular FSG.




$deg(v_1) = 0.8 + 0.9 + 0.6 = 2.3$
$deg(v_2) = 0.8 - (0.8 + 0.4) = -0.4$
$deg(v_3) = 0.6 - (0.4 + 0.4) = -0.2$
$deg(v_4) = 0.9 - (0.8 + 0.4) = 0.3$

## 5. TOTALLY VERTEX IRREGULAR FUZZY SIGNED GRAPH

If there is a vertex which has vertices with distinct totally total degree.

## 6. NEIGHBOURLY TOTAL VERTEX IRREGULAR FUZZY SIGNED GRAPH

A fuzzy signed graph is referred to as NTIFSG if each of its two adjacent vertices has a unique total degree.

**Theorem:**

Let Γ be a FSG where $\mu$ is constant function then Γ is neighbourly irregular if and only if Γ is a NTIFSG.

**Proof:**

Let Γ be a FSG and $v_1, v_2$ be two adjacent vertices in Γ.
Since Γ is NFSG so the degree of $v_1, v_2$ is distinct.
Suppose $\theta_1, \theta_2$ be the degrees of $v_1, v_2$ and $\theta_1 \neq \theta_2$.
We have to show that the total degree of these two nodes $t\ deg\ v_1, t\ deg\ v_2$ are distinct.
If possible $t\ deg\ v_1 = t\ deg\ v_2$

$$deg\ v_1 + \mu(v_1) = deg\ v_2 + \mu(v_2)$$
$$\theta_1 + \mu(v_1) = \theta_2 + \mu(v_2)$$
$$\theta_1 + C = \theta_2 + C \text{ ( is constant function)}$$
$$\theta_1 - \theta_2 = 0$$
$$\theta_1 = \theta_2$$

This is contradicts that $\theta_1, \theta_2$ are distinct.
Hence Γ is NTIFSG.

**Conversely,**

Let Γ is NTIFSG. Let $v_1, v_2$ be two adjacent vertices in Γ.
Since Γ is NTIFSG therefore $t\ deg\ v_1 \neq t\ deg\ v_2$.
Suppose $\theta_1, \theta_2$ be the degrees of $v_1$ and $v_2$.
Now,

$$t\ deg\ v_1 \neq t\ deg\ v_2$$
$$deg\ v_1 + \mu(v_1) \neq deg\ v_2 + \mu(v_2)$$
*So* $$\theta_1 + C \neq \theta_2 + C \text{ ( is constant function)}$$
$$\theta_1 - \theta_2 \neq 0$$
$$\theta_1 \neq \theta_2$$

Hence Γ is NIFSG

## 7. CONCLUSION

In this Paper we conclude that vertex regular and irregular fuzzy signed graphs Provide a rich framework for examining compare systems in which uncertainty and dualities are important on the hand vertex irregular fuzzy signed graphs where vertex degrees vary are more difficult to analyze than vertex regular fuzzy signed graphs where every vertex has the same degree allowing for a uniform distribution of relationship across the graph .Describe more diverse system in which some vertices play central or peripheral roles.

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